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Exact propagators for atom–laser interactions

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Abstract
A class of exact propagators describing the interaction of an N-level atom with a set of on-resonance δ-lasers is obtained by means of the Laplace transform method. State-selective mirrors are described in the limit of strong lasers. For the two-level case, the transient effects arising as a result of the interaction between both a semi-infinite beam and a wavepacket with the on-resonance laser are examined.

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The spacetime propagator can be considered as one of the most important tools in quantum physics since it governs any dynamical process. However, the knowledge of propagators corresponding to non-quadratic Hamiltonians is severely restricted. In this line, the spacetime propagator for a δ-potential relevant to tunnelling problems has excited much attention [1–5]. Such interactions turn out to be particularly useful to gain physical insight in systems where only integrated quantities are to be considered. A thorough discussion of point interactions as solvable models using a functional approach can be found in [6], and a formalism to incorporate general point interactions and dealing with different boundary conditions has been developed by Grosche [7, 8]. Even though the method is particularly suitable for calculating the energy-dependent Green function, a wide class of propagators was derived in such a fashion. The incorporation of time-dependent point interactions has been possible through different approaches as Duru's method [9] or the use of integrals of motion [10]. However, most of the effort has been focused on the dynamics of structureless particles and to the knowledge of the authors no attention has been paid to problems involving internal levels. Such a state of affairs contrasts dramatically with the current surge of activity in atom optics.

In this paper we use the method of Laplace transform [4, 5] to tackle particles with internal structure. In particular, we shall focus on exact propagators for atom–laser interactions, namely, those of an atom interacting with a set of δ-laser on-resonance with given interatomic transitions. The method is introduced in section 1 to obtain the exact propagator for a two-level atom. Details of the calculations relevant to the following section are here provided. The general case in which a given state is coupled to an arbitrary number of levels is discussed in section 2 where the high-intensity limit of the laser is related to state-selective mirrors.

Sections 3 and 4 respectively deal with the dynamics of a semi-infinite beam and a wavepacket of two-level atoms interacting with an on-resonance δ-laser. Such kinds of systems presents manifold applications in laser coherent control techniques such as cold atomic cloud compression [11], atom mirrors and beam splitters [12], and different schemes where fast transitions are required as in the implementation of logic gates for ion trap quantum computing [13]. Moreover, idealized time-of-arrival measurements [14] and recently proposed improvements in Ramsey interferometry with ultracold atoms [15] rest in a full quantum mechanical treatment of the dynamics of such systems.

1. The two-level atom

In this section, we use the method of Laplace transform to obtain the propagator for a two-level atom incident on a narrow perpendicular on-resonance laser beam. Spontaneous decay is assumed to be negligible throughout the paper and we shall consider effective one-dimensional systems in which the transverse momentum components can be neglected as is the case for atoms in narrow waveguides [16]. In a laser adapted interaction picture, and using the rotating wave approximation, the Hamiltonian describing the system is

$$\mathbf{H}_c = \frac{\widehat{p}^2}{2m}\mathbf{1}_2 + \mathbf{V}\delta(\widehat{x}-\xi) = \frac{\widehat{p}^2}{2m}\mathbf{1}_2 + \frac{\hbar\Omega}{2}\delta(\widehat{x}-\xi)\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \tag{1}$$

where $\hat{p}$ is the momentum operator conjugate to $\hat{x}$, the ground state $|1\rangle$ is in vector-component notation $\binom{1}{0}$, and the excited state $|2\rangle$ is $\binom{0}{1}$. The second term in the right-hand side defines the potential strength matrix $\mathbf{V}$ and $\mathbf{1}_2$ the two-dimensional identity matrix. Equation (1) may be regarded as the $\epsilon \to 0$ limit of a laser of width ϵ and Rabi frequency Ω_L keeping $\Omega = \Omega_L\epsilon$ constant. Ω_L here and in the following is chosen to be real. We start then by considering the free propagator for a one-channel problem on a Hilbert space of square integrable functions $\mathfrak{H}$ (see for instance [8]),

$$K_0(x,t|x',0) = \sqrt{\frac{m}{2\pi \mathrm{i}\hbar t}}\exp\left(\frac{\mathrm{i}m(x-x')^2}{2t\hbar}\right). \tag{2}$$

In what follows we shall be interested in describing the dynamics of particles with two internal levels. The free propagator ($\Omega = 0$) for states on the Hilbert space $\mathfrak{H}\otimes\mathbb{C}^2$ is given by $\mathbf{K}_0(x,t|x',t') = K_0(x,t|x',t')\mathbf{1}_2$, in the same interaction picture than (1). Moreover, δ-type of perturbations can be generally taken into account using the method of Laplace transform [5] which assumes the unperturbed propagator to be known. More precisely, the full propagator can be related to the free one through the Lippmann–Schwinger equation [4, 5]

$$\mathbf{K}(x,t|x',t') = \mathbf{K}_0(x,t|x',t') - \frac{\mathrm{i}}{\hbar}\int_{t'}^{t}\mathrm{d}t''\int_{-\infty}^{\infty}\mathrm{d}x''\,\mathbf{K}_0(x,t|x'',t'')\mathbf{V}(x'',t'')\mathbf{K}(x'',t''|x',t'). \tag{3}$$

Given that the potential has the form of a point interaction, the integral over x'' coordinates is straightforward. One can then take the Laplace transform with respect to t, which we denote with a tilde,

$$\widetilde{\mathbf{K}}(x,s|x',0) = \widetilde{\mathbf{K}}_0(x,s|x',0) - \frac{\mathrm{i}V_0}{\hbar}\begin{pmatrix} 0 & \widetilde{K}_0(x,s|\xi,0) \\ \widetilde{K}_0(x,s|\xi,0) & 0 \end{pmatrix}\widetilde{\mathbf{K}}(\xi,s|x',0), \tag{4}$$

where $V_0 = \hbar\Omega/2$ and we have made use of the convolution theorem $\left(\mathcal{L}\left[\int_0^t g(t-t')f(t')\,\mathrm{d}t'\right] = \widetilde{g}(s)\widetilde{f}(s)\right)$. By setting $x = \xi$ it is explicitly found that

$$\widetilde{\mathbf{K}}(\xi,s|x',0) = \begin{pmatrix} 1 & \frac{\mathrm{i}V_0}{\hbar}\widetilde{K}_0(0,s|00) \\ \frac{\mathrm{i}V_0}{\hbar}\widetilde{K}_0(0,s|00) & 1 \end{pmatrix}^{-1}\widetilde{\mathbf{K}}_0(\xi,s|x',0). \tag{5}$$

Next, we note the exact expression for the Laplace transform of the single-channel free propagator (2),

$$\widetilde{K}_0(x, s|x', 0) = \sqrt{\frac{m}{2\mathrm{i}\hbar s}}\, \exp\left(-\sqrt{\frac{2ms}{\mathrm{i}\hbar}}|x - x'|\right), \tag{6}$$

which becomes necessary for evaluating the inverse of the matrix. Combining (4) and (5) the Laplace transform of the exact full propagator is obtained, and taking the inverse transform one can find the spacetime propagator

$$\mathbf{K}(x, t|x', 0) = \mathbf{K}_0(x, t|x', 0) - \frac{\mathrm{i}V_0}{\hbar}\begin{pmatrix} I & J \\ J & I \end{pmatrix}, \tag{7}$$

with

$$\begin{aligned} I &= \frac{m}{2\mathrm{i}\hbar}\mathcal{L}^{-1}\left(\frac{\exp\left(-\sqrt{\frac{2ms}{\mathrm{i}\hbar}}(|x-\xi| + |\xi - x'|)\right)}{s - \mathrm{i}\frac{mV_0}{2\hbar^3}}\right), \\ J &= \frac{m}{2\mathrm{i}\hbar}\mathcal{L}^{-1}\left(-\mathrm{i}\frac{V_0}{\hbar}\sqrt{\frac{m}{2\mathrm{i}\hbar s}}\frac{\exp\left(-\sqrt{\frac{2ms}{\mathrm{i}\hbar}}(|x-\xi| + |\xi - x'|)\right)}{s - \mathrm{i}\frac{mV_0}{2\hbar^3}}\right). \end{aligned} \tag{8}$$

Fortunately in our case, the resulting matrix element can be related after taking partial fractions with standard results [17] so that

$$\begin{aligned} \mathbf{K}(x, t|x', t') = \mathbf{K}_0(x, t|x', t') &- \frac{mV_0}{4\hbar^2}\sum_{\alpha=\pm1}\exp\left(\frac{\alpha m V_0(|x-\xi| + |\xi - x'|)}{\hbar^2} + \mathrm{i}\frac{mV_0^2 t}{2\hbar^3}\right) \\ &\times \operatorname{erfc}\left[\alpha\sqrt{\frac{\mathrm{i}mV_0^2 t}{2\hbar^3}} + \frac{1}{2}\sqrt{\frac{2m}{\mathrm{i}\hbar t}}(|x-\xi| + |\xi - x'|)\right]\begin{pmatrix} \alpha & 1 \\ 1 & \alpha \end{pmatrix}. \end{aligned} \tag{9}$$

A more compact expression can be obtained by rewriting the full propagator in terms of the Moshinsky function (see appendix A),

$$\mathbf{K}(x, t|x', t') = \mathbf{K}_0(x, t|x', t') - \frac{mV_0}{2\hbar^2}\sum_{\alpha=\pm1} M\left(|x-\xi| + |\xi - x'|, \alpha\kappa, \frac{\hbar t}{m}\right)\begin{pmatrix} \alpha & 1 \\ 1 & \alpha \end{pmatrix}, \tag{10}$$

with $\kappa = -\mathrm{i}mV_0/\hbar^2$ for short. One should note that this propagator opens up the way to a whole variety of problems involving quantum dynamics of two-level atoms. A most relevant fact is that as long as the full Hamiltonian can be written as a direct sum, the same property holds for the propagator. Therefore, if $\mathbf{H} = \bigoplus_{s=1}^{d}\mathbf{H}_s$,

$$\mathbf{K}(x, t|x', t') = \bigoplus_{s=1}^{d}\mathbf{K}_s(x, t|x', t'), \tag{11}$$

which becomes very useful when one wishes to study the dynamics in a given subspace. In sections 3 and 4 we shall focus on some analytical examples dealing with the quantum dynamics of semi-infinite beams and wavepackets of two-level atoms.

2. The N-level atom

In this section we extend the previous approach to an N-level system living on $\mathfrak{H} \otimes \mathbb{C}^N$. Suppose that an N-level atom is subjected to the action of N on-resonance lasers ($\hbar\Omega_{ij}/2 = V_i$)

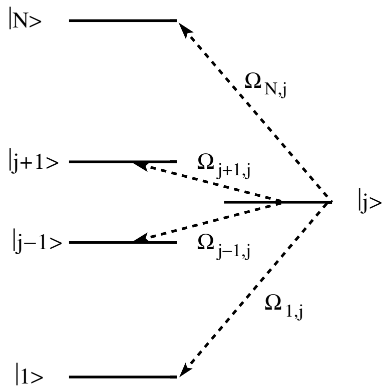

Figure 1. Configuration of an N-level atom interacting with an $(N-1)$ on-resonance fields coupling the $|j\rangle$ to any other level.

all of which are located at the same position and couple the $|j\rangle$ level $(1 \leqslant j \leqslant N)$ with the $(N-1)$ levels as shown in figure 1,

$$\mathbf{H}_{\rm c} = \frac{\widehat{p}^2}{2m}\mathbf{1}_N + \frac{\hbar}{2}\delta(\widehat{x}-\xi)\begin{pmatrix} 0 & \cdots & 0 & \Omega_{1,j} & 0 & \cdots & 0 \\ \vdots & & \vdots & \vdots & \vdots & & \vdots \\ 0 & \cdots & 0 & \Omega_{j-1,j} & 0 & \cdots & 0 \\ \Omega_{1,j} & \cdots & \Omega_{j-1,j} & 0 & \Omega_{j+1,j} & \cdots & \Omega_{N,j} \\ 0 & \cdots & 0 & \Omega_{j+1,j} & 0 & \cdots & 0 \\ \vdots & & \vdots & \vdots & \vdots & & \vdots \\ 0 & \cdots & 0 & \Omega_{N,j} & 0 & \cdots & 0 \end{pmatrix}. \tag{12}$$

Simplified configurations can be obtained just by setting certain coupling elements to zero. The exact propagator for a such system reads

$$\mathbf{K}(x,t|x',t') = K_0(x,t|x',t')\mathbf{1}_N - \frac{mV_m^{-1}}{2\hbar^2}\sum_{\alpha=\pm 1} M\left(|x-\xi|+|\xi-x'|, -\mathrm{i}\alpha m\frac{V_m}{\hbar^2}, \frac{\hbar t}{m}\right)$$
$$\times\begin{pmatrix} \alpha V_1^2 & \cdots & \alpha V_1V_{j-1} & V_1V_m & \alpha V_1V_{j+1} & \cdots & \alpha V_1V_N \\ \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \alpha V_{j-1}V_1 & \cdots & \alpha V_{j-1}^2 & V_{j-1}V_m & \alpha V_{j-1}V_{j+1} & \cdots & \alpha V_{j-1}V_N \\ V_1V_m & \cdots & V_{j-1}V_m & \alpha V_m^2 & V_{j+1}V_m & \cdots & V_NV_m \\ \alpha V_{j+1}V_1 & \cdots & \alpha V_{j+1}V_{j-1} & V_{j+1}V_m & \alpha V_{j+1}^2 & \cdots & \alpha V_{j+1}V_N \\ \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \alpha V_NV_1 & \cdots & \alpha V_NV_{j-1} & V_NV_m & \alpha V_NV_{j+1} & \cdots & \alpha V_N^2 \end{pmatrix}, \tag{13}$$

where $V_m = \sqrt{\sum_{i\neq j} V_i^2}$. Incidentally, the ladder configuration in which only successive levels are coupled to each other does not admit a similar generalization to N-level given the complexity of the inverse matrix. However, for the three-level case one can find the ladder, V and Λ configurations using equation (13). One may then study the spacetime dynamics of coherent trapping which results from the destructive quantum interferences between the two transitions, for initial states in a superposition of the two lower levels $|1\rangle$ and $|2\rangle$ [18].

2.1. Intense lasers and propagators for state-selective mirrors

For the one-channel case, the effect of an infinite strength of the δ-potential is tantamount to a hard-wall boundary condition [5, 19]. This feature has been extensively exploited in the exact perturbation theory developed by Grosche to obtain a wide class of Green functions and propagators including boundaries (say, in half-space or boxes) [7, 8]. In atom optics, it is also well known that a very intense laser behaves as a totally reflective mirror for the states coupled by it. Therefore, we shall consider the limit in which one of the lasers is made infinitely strong. In order to do so, it is convenient to note that the following relation holds [5]:

$$\lim_{V_n\to\infty} \frac{mV_n}{\hbar^2} M\left(|x-\xi|+|\xi-x'|, -\mathrm{i}\alpha m\frac{V_m}{\hbar^2}, \frac{\hbar t}{m}\right) = \alpha K_0(|x-\xi|+|\xi-x'|, t|0,0).$$

The general expression for the propagators with N-lasers in which the nth one (coupling the states $|j\rangle$ and $|n\rangle$) is made infinitely strong becomes diagonal,

$$[\mathbf{K}(x,t|x',t')]_{ik} = K_0(x,t|x',t')\delta_{ik} - K_0(|x-\xi|,t||x'-\xi|,t')\delta_{ik}(\delta_{jk}+\delta_{nk}), \tag{14}$$

δ_{ij} being the Krönecker delta. The laser behaves then as a totally reflecting mirror selective to the states coupled by it, and suppresses for such states any possible excitation due to the presence of other lasers located at the same position.

Another interesting case is the on-resonance excitation of a couple of levels with high-intensity lasers. This leads to the limit where the strength coefficients of the two lasers go to infinite, $V_l, V_n \to \infty$ and $n > l$, but its ratio is kept constant, $V_l/V_n = c$ with $c > 0$. For such a case one finds

$$\begin{aligned}[\mathbf{K}(x,t|x',t')]_{ik} = {} & -K_0(|x-\xi|,t||x'-\xi|,t')\frac{c}{\sqrt{1+c^2}}(\delta_{ni}\delta_{lk}+\delta_{li}\delta_{nk}) + \delta_{ik}\Bigg[K_0(x,t|x',t') \\ & - K_0(|x-\xi|,t||x'-\xi|,t')\left(\delta_{kj}+\frac{c^2}{\sqrt{1+c^2}}\delta_{lk}+\frac{1}{\sqrt{1+c^2}}\delta_{nk}\right)\Bigg].\end{aligned} \tag{15}$$

The difference now arises from the fact that the state-selective mirrors have a finite reflectivity for states $|l\rangle$ and $|n\rangle$, to which excitation is allowed. However, note that for the $|j\rangle$ state the laser still mimics a totally reflecting mirror.

3. Moshinsky shutter

We next study a time-dependent multi-channel scattering problem and consider a monochromatic beam of two-level atoms in its ground-state incident on a totally absorbing shutter which is suddenly removed at time $t = 0$. Such a kind of set-up is usually referred to as a Moshinsky shutter, ever since the seminal paper [20] which led to the discovery of diffraction in time. The conditions on the reflectivity of the shutter can be easily modified to more general cases [20, 21]. The initial state is then of the form

$$\Psi(x, t=0) = \mathrm{e}^{\mathrm{i}kx}\Theta(-x)|1\rangle, \tag{16}$$

where $\Theta(x)$ is the Heaviside step function. Equation (16) is an obvious generalization of the Moshinsky type of initial condition for a single-channel problem, discussed in the context of diffraction in time. We note that such a kind of state is not normalizable in the usual sense, yet accurately describes certain experimental set-ups [22] and provides a basis for the wavepacket analysis.

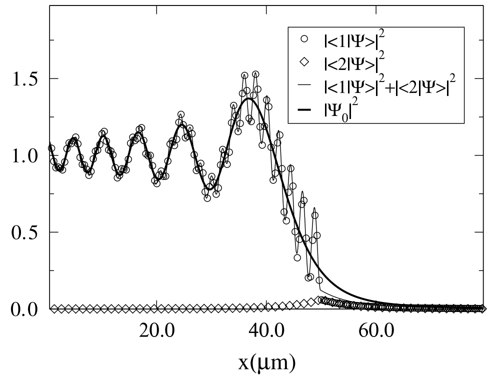

Figure 2. Probability density of the ground and excited states, and total probability density in the presence of the laser (located at $\xi = 50\ \mu$m) and for the free case, 50 ms after removing the shutter initially located at the origin. The incident beam moves at $v = 0.1$ cm s^{-1} (mass of ^{87}Rb). The picture is taken at the instant in which the classical profile (the step function $\Theta(vt - x)$) reaches the laser.

The time evolution can be studied using the superposition principle. If we consider first the free evolution, with $\Omega = 0$, then the result is that of diffraction in time,

$$\begin{aligned}\Psi_0(x,t) &= \int_{-\infty}^{\infty} \mathrm{d}x' \mathbf{K}_0(x,t|x',t')\Psi(x,t=0)\\ &= M(x,k,\hbar t/m)|1\rangle.\end{aligned} \tag{17}$$

This solution was found by Moshinsky and has been observed in a wide variety of experiments with ultracold neutron interferometry for the one-channel case [22]. It is a well-known fact that it tends with increasing time to the stationary wavefunction. We further note that due to the absence of coupling between the internal states in $\mathbf{K}_0(x,t|x',t')$, the excited state $|2\rangle$ is not populated, in agreement with (11).

An alternative configuration in which the beam tunnels through a δ-barrier was discussed in [4, 23, 24].

Let us now look at the time evolution in the presence of the laser. Using the integral (A.4) in the appendix, the exact solution can be found in close form,

$$\begin{aligned}\Psi(x,t) = \int_{-\infty}^{\infty} \mathrm{d}x'\, \mathbf{K}(x,t|x',t')\Psi(x,t=0) = \Psi_0(x,t) + \frac{1}{2}\sum_{\alpha=\pm1}\begin{pmatrix}\alpha\\1\end{pmatrix}\frac{\kappa}{k-\alpha\kappa}\\ \times\{[M(|x-\xi|+\xi,k,\hbar t/m) - M(|x-\xi|+\xi,\alpha\kappa,\hbar t/m)]\}.\end{aligned} \tag{18}$$

The total probability density is plotted in figure 2. The velocity of the incident beam is chosen in all simulations to satisfy $v = \hbar q/m = V_0/\hbar$, for which one finds, after solving the two-channel stationary problem, that the reflection and transmission probabilities in the excited state, $|R_2|^2 = |T_2|^2$, are maximized and indeed equal those in the ground state, $|R_1|^2 = |T_1|^2 = 1/4$. The paradigmatic oscillations on the probability density, main feature of the diffraction in time described by the Moshinsky function ($|\Psi_0|^2$), is modified for all $x < \xi$ due to the interference which arises with the reflected part. Indeed, for later times such interference completely dominates and the density profile is dramatically perturbed as shown in figure 3.

In figure 4 we plot the time evolution of the probability density in the excited state $|2\rangle$. The delta laser is shown to behave as a point-like source of atoms. Moreover, the pattern exhibits diffraction in both time and space domain.

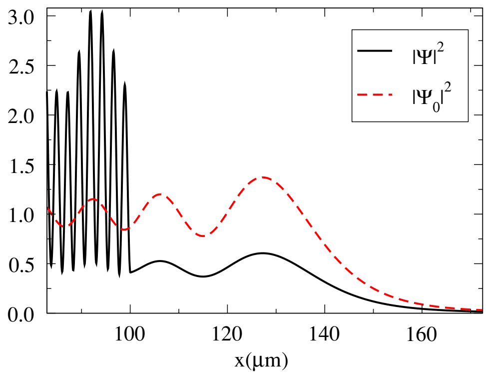

Figure 3. Total probability density for an incident beam in the ground state in the presence of the laser (located at $\xi = 100\ \mu$m) and for free space, 150 ms after removing the shutter initially located at the origin. The incident beam moves at $v = 0.1$ cm s^{-1} (mass of ^{87}Rb), exhibiting interference for all $x < \xi$, as a result of the reflexion from the laser.

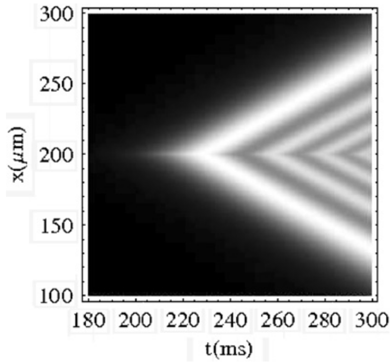

Figure 4. Spacetime density plot of the population in the excited state, $|\langle 2|\Psi\rangle|^2$, with $v = 0.1$ cm s^{-1}, and the position of the laser $\xi = 200\ \mu$m. The grey scale changes from dark to light as the function values increase.

4. Wavepacket dynamics

Many experiments deal with finite samples rather than beams. In the one-channel case, the tunnelling dynamics of wavepackets through narrow barriers have been examined in a series of works [4, 19, 25]. In addition, the phenomenon of quantum deflection was predicted in the presence of semi-transparent and perfect mirrors [26–28].

We next consider normalizable states belonging to $\mathcal{L}^2(\mathbb{R}) \otimes \mathbb{C}^2$. In particular we study the dynamics of eigenstates of a hard-wall trap which are released at time $t = 0$ and launched with momentum $\hbar q$ against the on-resonance delta laser. All-optical box traps have been recently been obtained in the laboratory [29]. More precisely we assume an initial sine-wavepacket given by

$$\Psi(x, t = 0) = \frac{1}{2\mathrm{i}}\sqrt{\frac{2}{L}} \sum_{\beta=\pm 1} \beta\, \mathrm{e}^{\mathrm{i}q_{n\beta}x} \chi_{[0,L]} |1\rangle \tag{19}$$

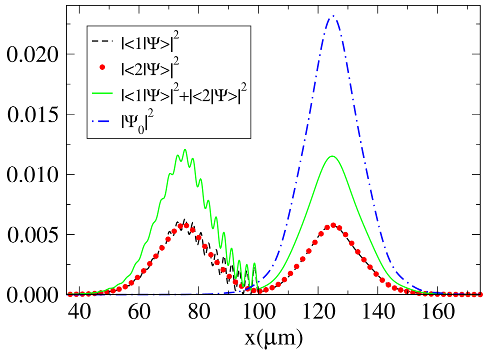

Figure 5. Ground, excited and total probability densities for an incident sine-wavepacket ($n = 1$) released at $t = 0$ from a hard-wall trap of size $L = 50\,\mu$ m and centred at 25 μm from the origin, with $q = mV_0/\hbar^2$ and velocity 0.1 cm s^{-1}, after 100 ms of evolution. The position of the laser is $\xi = 100\,\mu$m. The freely time-evolved sine wavepacket is also plotted.

with $q_{n\beta} = q + \beta n\pi/L$. The expansion of such a state has recently been discussed at the single-channel level in free space [30], in the presence of gravity [31], and when generalized to the many-body Tonks–Girardeau regime [32]. For the interaction with the on-resonance delta laser one can actually propagate in time this initial condition. The time-evolved wavefunction is

$$\begin{aligned}\Psi_0(x,t) &= \int_{-\infty}^{\infty} \mathrm{d}x' K_0(x,t|x',t=0)\Psi(x',t'=0)\\ &= \frac{1}{4\mathrm{i}}\sqrt{\frac{2}{L}}\sum_{\beta=\pm1}\beta[\mathrm{e}^{\mathrm{i}q_{n\beta}L}M(x-L,q_{n\beta},\hbar t/m) - M(x,q_{n\beta},\hbar t/m)]|1\rangle \end{aligned} \tag{20}$$

and,

$$\begin{aligned}\Psi(x,t) = \Psi_0(x,t) + \frac{1}{4\mathrm{i}}\sqrt{\frac{2}{L}}\sum_{\alpha,\beta=\pm1}\begin{pmatrix}\alpha\\1\end{pmatrix}\frac{\beta\kappa}{k-\alpha\kappa}\{\mathrm{e}^{\mathrm{i}p_\beta L/\hbar}[M(|x-\xi|+\xi-L,k,\hbar t/m)\\ -M(|x-\xi|+\xi-L,\alpha\kappa,\hbar t/m)] - [M(|x-\xi|+\xi,k,\hbar t/m)\\ -M(|x-\xi|+\xi,\alpha\kappa,\hbar t/m)]\}.\end{aligned} \tag{21}$$

Figure 5 shows the dynamics when the incident momentum is such that maximum excitation is achieved. The laser acts as a beam splitter dividing the wavepacket into two parts, the transmitted one being similar in shape to the freely evolving wavepacket, whereas the reflected part exhibits interference. Note that (18) and (21) admit a simple generalization for the respective N-level problem. In order to do so it suffices to consider the suitable momenta and prefactor of the matrix in the propagator for each of the channels.

5. Discussion

We have generalized the method of Laplace transform to include point-like perturbations in the dynamics of particles with internal structure. In such a fashion we have obtained the spacetime propagator for an N-level atom interacting with a set of on-resonance delta lasers. For strong lasers, the propagators for state-selective mirrors have been obtained. A similar procedure could be applied to the Green function for which an exact perturbation theory has been developed and extensively discussed by Grosche [8]. The inclusion of time dependence in the propagators to adiabatically turn the lasers on and off independently from each other is an open problem which would provide access to the spacetime dynamics of many relevant phenomena in quantum optics. The dynamics of a semi-infinite beam and a wavepacket of two-level atoms incident on the laser, with a straightforward generalization for the N-level case, have also been worked out.

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Appendix. The Moshinsky function

The Moshinsky function arises in most of the problems where 1D quantum dynamics involves sharp boundaries well in time or space domains. Similarly it is found when considering free propagators perturbed with point interactions.

Its standard definition reads

$$M(x,k,\tau) := \frac{e^{i\frac{x^2}{2\tau}}}{2} w(-z), \tag{A.1}$$

where

$$z = \frac{1+i}{2}\sqrt{\tau}\left(k - \frac{x}{\tau}\right), \tag{A.2}$$

and the so-called Faddeyeva function [33] w is explicitly defined as

$$w(z) := e^{-z^2}\mathrm{erfc}(-iz) = \frac{1}{i\pi}\int_{\Gamma_-} du \frac{e^{-u^2}}{u-z}, \tag{A.3}$$

Γ_- being a contour in the complex z-plane which goes from $-\infty$ to ∞ passing below the pole. After [20], $M(x,k,\tau)$ has been named the Moshinsky function.

For the exact time evolution in the presence of a laser with a cut-off plane wave or hard-wall eigenstates as initial conditions, we find integrals of the form

$$\int^x dx' e^{ikx'} M(ax'+b,c,\tau) = \frac{e^{ikx}}{i(k+ca)}[M(ax+b,c,\tau) - M(ax+b,-k/a,\tau)]. \tag{A.4}$$

For more details we refer the reader to [4].

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